01 SIMP LIFICATION AND APPROXIMATION

Some ways of simplifying calculations

1. **ADDITION:**

342 + 557 + 629 + 746 + 825 = ?
When we are adding three-digit numbers, first add two-digits at a time (units and tens place).
42 + 57 + 29 + 46 + 25 = 199.
To add 42 and 57, mentally treat 57 as 50 + 7 (50 would facilitate quick addition).
Thus 42 + 57 (42 + 50) + 792 + 799.
Similarly, 99 + 29 = (99 + 20) + 9 = 128.
128 + 46 = (128 + 40) + 6 = 174.
174 + 25 = (174 + 20) + 5 = 199.
The last two digits (the units place and the tens place) of the addition are 99, while. the digit I is to be carried forward).
Now add
1(carried) + 3 + 5 + 6 +7 + 8 = 30.
∴ The Answer is 3099.
Note: -The same logic can be extended to four-digit additions: **SUBTRACTION:**
987 – 256 =?

2. Instead of taking a single digit at a time, subtractions would be faster by taking two digits i.e.
87 – 56 = 31.
900 – 200 = 700
∴ The Answer is 731

3. For multiplication by 5, you should multiply the figure given by 10 and then divide it by 2.
Example: 649 × 5 = $\frac{6490}{2}$ = 3245.

4. For multiplication by 25, you should multiply the figure given by 100 and divide it by 4.
Example: 643 × 25 = $\frac{64300}{4}$ = 16075.

5. For multiplication by 125, you should multiply the figure given by 1000 and divide by 8.
Example: 492 × 125 = = $\frac{49200}{8}$ = 6150.

6. For multiplication by 11, the rule is "for each digit add the right-hand digit and write the result as the the corresponding figure in the product". For purpose of applying the rule, it will be easier, if you assume that there is one "zero" on either side of the given number. **Example:** 3569 × 11 ⟶ 0|3569|0 ⟶ 39259.

7. For multiplication by 12, the rule is "simply multiply it by 10 first and then add the number to the double of it.
Example: 756 × 12 ⟶ 7560 + 1512 = 9072.

8. For multiplication by 13, the rule is "three times each digit added to the right-hand digit gives the corresponding digit in the product". e.g. 0|92856|0 x 13 = 1207128.

9. Multiplication by 19, can be treated as multiplication by (20 – 1); e.g. 92856 × 19 = 92856 × 20 – 92856 = 1764264

10. Method to multiply 2-digit number.
AB × CD = AC / (AD + BC) / BD
(a) 35 × 47 = 12 / (21 + 20) / 35 = 12 / 41 / 35 = 1645
(b) 27 × 63 = 12 / (6 + 42) / 21 = 12 / 48 / 21 = 1701

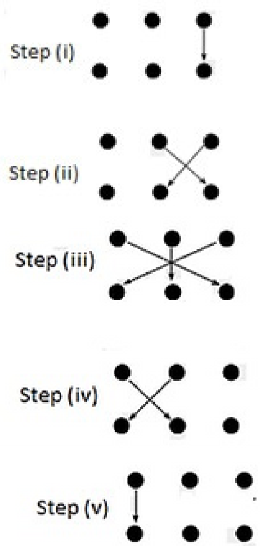
11. Method to multiply 3-digit no.

ABC × DEF = AD / (AE + BD) / (AF + BE + CD) / (BF + CE) / CF

(a) 456 × 234 = 4 × 2 / (4 × 3 + 5 × 2) / (4 × 4 + 5 × 3 + 6 × 2) / (5 × 4 + 6 × 3) / 6 × 4
= 8 / 12 + 10 / 16 + 15 + 12 / 20 + 18 / 24
= 8 / 22 /43 / 38 / 24 = 106704

(a) 257 × 843 = 2 × 8 / (2 × 4 + 5 × 8) / (2 × 3 + 5 × 4 + 7 × 8) / (5 × 3 + 7 × 4) / 7 × 3
= 16 / (8 + 40) / (6 + 20 + 56) / (15 + 28) / 21
= 16/ 48/82/ 43 / 21 = 216651

The following steps, we are performing for multiplication:



1. Those number whose all digits are 1.

A number whose one's, ten's, hundred's digit is 1 i.e., 11, 111, 1111, ...

In this we count number of digits. We write 1, 2, 3, n their square the digit in the number, then write in decreasing order up to 1.

112 = 121

1112 = 12321

11112 = 1234321

2. Those numbers whose all digits are 3.

(33)2 = 1089 Those number in which all digits are number is 3 two or more than 2 times repeated, to find the square of these number, we repeat 1 and 8 by (n – 1) time. Where n → Number of times 3 repeated. (333)2 = 110889 (3333)2 = 11108889

3. Those number whose all digits are 9.

(99)2 =9801

(999)2 = 998001

(9999)2 = 99980001

(99999)2 = 9999800001

4. If in a series all number contains repeating 7. To find their sum, we start from the left multiply 7 by 1, 2, 3, 4, 5 & 6. Look at the example below.

777777 + 77777 + 7777 + 777 + 77 + 7 = ?

= 7 × 1 / 7 × 2 / 7 × 3 / 7 × 4 / 7 × 5 / 7 × 6

= 7 / 14/ 21 / 28 / 35 / 42= 864192

0.5555 + 0.555 + 0.55 + 0.5 = ?

5.

To find the sum of those number in which one number is repeated after decimal, then first write the number in either increasing or decreasing order. Then -find the sum by using the below method.

0.5555 + 0.555 + 0.55 + 0.5

= 5 × 4 / 5 × 3 / 5 × 2 / 5 × 1

= 20 / 15/ 10/ 5 = 2.1605

Perfect Square

Square Numbers 1 to 50

12 = 1	112 = 121	212 = 441	312 = 961	412 = 1681
22 = 4	122 = 144	222 = 484	322 = 1024	422 = 1764
32 = 9	132 = 169	232 = 529	332 = 1089	432 = 1849
42 = 16	142 = 196	242 = 576	342 = 1156	442 = 1936
52 = 25	152 = 225	252 = 625	352 = 1225	452 = 2025
62 = 36	162 = 256	262 = 676	362 = 1296	462 = 2116
72 = 49	172 = 289	272 = 729	372 = 1369	472 = 2209
82 = 64	182 = 324	282 = 784	382 = 1444	482 = 2304
92 = 81	192 = 361	292 = 841	392 = 1521	492 = 2401
102 = 100	202 = 400	302 = 900	402 = 1600	502 = 2500

Properties of square numbers:

1. The square of an even number is always an even number. E.g. 62 =36, 82 = 64, 142 = 256.
2. The square of an odd number is always an odd number. E.g. 92 = 81, 132 = 169, 232 = 529.
3. A perfect square can never end with 2,3,7,8. (a perfect square is square of an integer)
4. The number of zeros at the end of a perfect square is never odd. E.g. 10,000 is a perfect square but 1,000 is not.
5. The square of any natural number is equal to the sum of first n odd numbers. E.g.
   32 = 1 + 3 + 5 = 9
   52= 1 + 3 + 5 + 7 + 9 = 25
6. For any natural number N; N2 + N + (N+1) = (N+1)2
   E.g. 112 = 121; 122 = 144
   So, 112 + 11 +12 = 121 +11+12 = 144 = 122 = 122
7. Every perfect square is either a multiple of 3 or exceeds the multiple of 3 by 1.
8. Every perfect square is either a multiple of 4 or exceeds the multiple of 4 by 1.

Squares Fun Facts:

(2)2 = 1 + 2 + 1 = 4

(3)2 = 1 + 2 + 3 + 2 + 1 =9
(4)2 = 1 + 2 + 3 + 4 + 3 + 2 + 1= 16
(5)2 = 1 + 2 + 3 + 4 + S + 4 + 3 + 2 + 1= 25
(6)2 = 1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1= 36
(7)2 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1=49
(3)2 = 1 + 2 + 3 + 4 + 5 + 6+7 + S + 7 + 6 + 5 + 4 + 3 + 2 +1=64

(9)2 = 1 + 2 + 3 + 4 + 5 + 6+7 + 8 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2+1= 81

Square Root:

Aswenowknowthata=a×a;Leta be N. Then the square root of N = a.

We also know that (-a) × (-a) = a2. Therefore square root of N can also assume the value of -a.
So every number N has two square roots, one positive and the other negative.
E.g. Square root of 36 = +6, -6
The symbol '√' denotes root of a number.

Cubes:

a3 = a × a × a; Here a3 is called the third power of 'a', which is commonly refereed as cube of 'a'.
Given below is a table that illustrates squares and cubes of all natural numbers till 30 for your ready referral.

Number	Square	Cube	Number	Square	Cube
1	1	1	16	256	4096
2	4	8	17	289	4913
3	9	27	18	324	5832
4	16	64	19	361	6859
5	25	125	20	400	8000
6	36	216	21	441	9261
7	49	343	22	484	10648
8	64	512	23	529	12167
9	81	729	24	576	13824
10	100	1000	25	625	15625
11	121	1331	26	676	17575
12	144	1728	27	729	19683
13	169	2197	28	784	21952
14	196	2744	29	841	24389
15	225	3375	30	900	27000

Some properties of square root:

(i) Complete square of a no. is possible if its last digit is 0, 1, 4, 5, 6 & 9. If last digit of a no. is 2, 3, 7, 8 then complete square root of this no. is not possible.

(ii) If last digit of a no. is 1, then last digit of its complete square root is either 1 or 9.

(iii) If last digit of a no. is 4, then last digit of its complete square root is either 2 or 8.

(iv) If last digit of a no. is 5 or 0, then last digit of its complete square root is either 5 or 0.

(v) If last digit of a no. is 6, then last digit of its complete square root is either 4 or 6.

(vi) If last digit of a no. is 9, then last digit of its complete square root is either 3 or 7.

(vii) If last digit of a no. is 9, then last digit of its complete square root is either 3 or 7.

Perfect Cube

4 × 4 × 4 = 64

7 × 7 × 7 = 343

The numbers mentioned above are perfect cubes. As a result, if a natural number is a cube of another natural number, it is a perfect cube. In other terms, a natural number x is a perfect cube if its cube is y, x3= y.

Some properties of cube and cube root:

(i) All even natural number cubes are even.
(ii) All odd natural number cubes are odd.
(iii) The square of the sum of the cubes of the first n natural numbers is equal to their sum.
(iv) Numbers ending in the digits 4, 5, 6, and 9 are cubes of numbers ending in the same digit. The cubes of numbers that end with the digit 2 end with the digit 8. And the cubes of numbers that end with digits 8 ends with the digit 2. The cubes of numbers ending in three digits and seven digits end in three digits and seven digits, respectively.
(vi) Adding Consecutive Odd Numbers

1 = 1 = 13
3 + 5 = 8 = 23
7 + 9 + 11= 27 = 33
In the above pattern, adding consecutive odd numbers gives the sum that is further a cube of a natural number in the number series.

(vii) Patterns in triangular Numbers
While observing the sums of cubes numbers, it can be said that the square of the xth **triangular** number is equal to the sum of the first x cube numbers.
For example: The first cube number is 1 (13) which is also equal to 12
The sum of first two cube numbers = 13 + 23 = 1 + 8 = 9 = 32
The sum of first three cube numbers = 13 + 23 + 33 = 1 + 8 + 27 = 36 = 62
The sum of first four cube numbers = 13 + 23 + 33 + 43 = 1 + 8 + 27 + 64 = 100 = 102
And so on.

Operations on Real Numbers:

When a series of calculation or operation signs like ' + ,' '-', ' x ', ' ÷ ' are given, a precedence of operation is to be followed.

For example, 2+2÷2. If we do addition first and then division, then this question becomes (2+2)÷2=4÷2=2

If we do division first and then addition, then this question becomes 2+(2÷2) 2+1=3
We know this precedence popularly as-VBODMAS.
V ------------Vinculum
B ------------ Brackets-in the order , (1),[]
O ------------ Of
D ------------ Division
M ------------ Multiplication
A ------------ Addition
S ------------ Subtraction
Types of Brackets-there are three standard types of brackets: Parentheses ( ), Square brackets [ ], Curly brackets {}.

After we have done Vinculum calculation,
(i) 1st we do the bracket calculation.
(ii) Then we do the "of" calculation.
(iii) Next, we do division or multiplication. Division and multiply can be done interchangeably.
(iv) Finally, we do addition or subtraction. Addition and subtraction can be done interchangeably.

Do the following simplifications.
(a) 5+7×6
(b) 90−8×7
(c) 90÷10−5×1
(d) 65−3×4+3×2
(e) 40÷(2+2×(10−6))
(f) [9+{6(5+3×2-1)+6}]

Solution
(a) 1st we will do the multiplication and then addition

5+7×6=5+42=47

(b) 1st we will do the multiplication and then subtraction. 90−8×7=90−56=34
(c) This question involves three operations.
1st we will do division, then multiplication (or vice versa) and then subtraction.
90÷10−5×1=9−5=4
(d) 65−12+6=59
(e) 40÷(2+2×(10−6))=40÷(2+2×4)=40÷(2+8)=40÷10=4
(f) [9+ {6(5+3 × 2-1) +6}] = [9 + {6 (5 + 6 -1) + 6}] = [9 + {6 (10) + 6}] = [9 + 66] = 75

02 IMPORTANT FORMULAE / SHORTCUT METHODS

SIMPLE EQUATIONS AND RATIO PROPORTION VARIATION

(i) If a < b then (a + x): (b + x) > a:b

(ii) If a > b then (a + x): (b + x) < a:b

(iii) If a = b then (a + x):(b + x) = a:b

1. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ ...., then each of these ratios is equal to $\frac{a+c+e+\ldots\ldots}{b+d+f+\ldots\ldots}$

2.

(i) If $\frac{a}{K1} = \frac{b}{k2} = \frac{c}{K3}$ = then $\frac{a+b+c+\cdots}{c} = \frac{K1+K2+K3+\cdots}{K3}$

For example: If $\frac{P}{3} = \frac{Q}{4} = \frac{R}{7}$, then find $\frac{P+Q+R}{R}$

Sol.P=3,Q=4,R=7'

Then $\frac{P+Q+R}{R} = \frac{3+4+7}{7} = 2$

(ii) If $\frac{a_1}{a_2} = \frac{a_2}{a_3} = \frac{a_3}{a_4} = \frac{a_4}{a_5} \cdots \frac{a_n}{a_{n+1}}$ = k, then $a_1 : a_{n+1} = (k)^n$

3. A number added or subtracted from a, b, c & d, so that they are in proportion = $\frac{ad-bc}{(a+d)-(b+c)}$

For example: When a number should be subtracted from 2, 3, 1 & 5 so that they are in proportion. Find that number.

Sol. Req.No.= $\frac{2\times5-3\times1}{(2+5)-(3+1)} = \frac{10-3}{7-4} = \frac{7}{3}$

4. If X part of A is equal to Y part of B, then A: B = Y: X.

For example: If 20% of A = 30% of B, then find A: B

Sol. A:B= $\frac{30\%}{20\%} = \frac{3}{2}$ =3:2

5. If Ax = By = Cz, then A : B : C = $\frac{1}{x} : \frac{1}{y} : \frac{1}{z}$

6. If three qualities a, b and c are such that a: b :: b : c, then we say that they are in CONTINUED PROPROTION. We also get b^2 = ac. In such a case, c is said to be the third proportional of a and b. Also, b is said to be the mean proportional of a and c.

7.

(i) If X varies directly with Y and we have two sets of values of the variables X and Y – X1 corresponding to Y1 and X2 corresponding to Y2, then, since X ∝ Y, we canwrite down $\frac{X_1}{Y_1} = \frac{X_2}{Y_2}$ or $\frac{X_1}{X_2} = \frac{Y_1}{Y_2}$

(ii) If X varies inversely with Y and we have two sets of values of X and Y – X1 corresponding to Y1 and X2 corresponding to Y2, then since X and Y are inversely related to each other, we can write down X1Y1 = X2Y2 or $\frac{X_1}{X_2} = \frac{Y_2}{Y_1}$

(iii) If there are three quantities A, B and C such that A varies with B when C is constant and varies with C when B is constant, then A is said to vary jointly with B and C when both B and C are varying. i.e., A α B when C is constant and A α C when B is a constant; ⇒ A α BC

A α BC ⇒ A = kBC where k is the constant of proportionality.

8. Condition for solvability or consistency:

Let $a1x + b1y + c1 = 0$ and $a2x + b2y + c2 = 0$ be a system of simultaneous linear equations. The system can be tested for consistency according as

Test	Conclusion	Nature of Graph
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ or $\frac{c_1}{c_2}$	Consistent and Unique solution	Intersecting Lines
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Consistent and Infinite solution	Coincident Lines
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Inconsistent and No solution	Parallel Lines

9. Methods for solving simultaneous linear equations:

If the system is consistent and has unique solution then the equations can be solved by any of the following methods.

(a) **Method of elimination by substitution:**
In this method, we express one of the variables in terms of the other variable from either of the two equations and then this expression is put in the other equation to obtain an equation in one variable.

(b) **Method of elimination by equating the coefficients:**

In this method we eliminate one of the two variables to obtain an equation in one variable which can easily be solved. Putting the value of this variable in any one of the given equations, the value of the other variable can be obtained.

(c) **Method of cross-multiplication:**

Let a1 x + b1 y + c1 =0 and a2 x + b2 y + c2 = 0. Be a system of simultaneous linear equations in two variables x and y such that the system has a unique solution. Then the solution is given by

$$x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)} \text{ and } y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)}$$

REMARK: The above solution is generally written as

$$\frac{x}{(b_1c_2 - b_2c_1)} = \frac{y}{(c_1a_2 - c_2a_1)} = \frac{1}{(a_1b_2 - a_2b_1)}$$

Average, Weighted Average Fundamentals, Mixtures and Alligation

1. Average = $\frac{\text{Sum of all items in the group}}{\text{Number of items in the group}}$

2. The general form of weighted average is

$$Awt = \frac{(A1 \times wt1) + (A2 \times wt2) + (A3 \times wt3) + ... + (An \times wtn)}{wt1 + wt2 + wt3 + ... + wtn},$$

Where

Awt, is the weighted average of A1, A2, A3, ...An having weights wt1, wt2, wt3 ....wtn, respectively.

3. $Awt = \frac{(A1 \times w1) + (A2 \times w2)}{w1 + w2}$

⇒ (Awt ×w1)+(Awt ×w2) = (A1 × w1)+(A2×w2)

⇒ (Awt×w1)-(A1×w1) =(A2×w2)-(Awt ×w2)

⇒ W1×(Awt-A1) = w2×(A2-Awt)

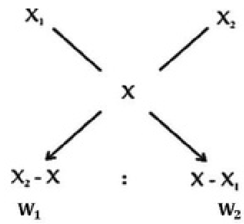
⇒ $\frac{w1}{w2} = \frac{(A_2 - A_{wt})}{Awt - A1}$

This is the Alligation formula.

The ratio of the weights of the two items mixed will be inversely proportion to the deviation of attributes of these two items from the average attribute of the resultant mixture.

$\frac{w1}{w2} = \frac{(x_2 - x)}{(x - x_1)}$

Allegation Cross:



Alligation does not give the actual volumes to be mired but only the ratio in which volumes are to be mixed

4. If there is P volume of pure liquid initially and in each operation, Q volume is taken out and replaced by Q volume of water, then at the end of n such operations, the concentration (k) of the liquid in the solution is given by = $\{\frac{P - Qn}{P}\}$ = k

This gives the concentration (k) of the liquid as a PROPORTION of the total volume of the solution.

Percentage, Profit Loss, Partnership, and Simple and Compound Interest

1.

(i) If Ais(x%=) $\frac{a}{b}$ more than B, then B is ($\frac{a}{a+b}$%) less than A.

(ii) If Ais(x%=) $\frac{a}{b}$ less than B, then B is ($\frac{a}{a+b}$%) more than A.

if a>b,we take a−b

if b<a,we take b−a

2. If price of a article increase from ₹ a to ₹ b, then its expenses decrease by ($\frac{b-a}{b}$ ×100)% so that expenditure will be same.

Due to increase/decrease the price by x%, A man purchase a kg more in ₹ y, then

Per Kg increase or decrease = ($\frac{xy}{100 \times a}$)

Per Kg starting price = ₹($\frac{xy}{100 \pm x)a}$)

For two articles, if price :

3.

Ist	IInd	Overall
Increase (x%)	Increase (y%)	Increase (x+y+ $\frac{xy}{100}$)%
Increase (x%)	Decrease (y%)	(x−y− $\frac{xy}{100}$)% If+ve (Increase) If−ve (decrease)
Decrease (x%)	Decrease (y%)	Decrease (x+y− $\frac{xy}{100}$)%
Increase (x%)	Decrease (x%)	Decrease ($\frac{x2}{100}$)%

4. If the side of a square or radius of a circle is x% increase/decrease, then its area increase/decrease =

$(2x \pm \frac{x^2}{100})\%$

Ifthe sideofasquare, x% increase/decrease then x% its perimeter and diagonal increase/decrease

5.

(i) If population P increase/decrease at r% rate, then after t years population = $p(\frac{100 \pm R}{100})^t$

(ii) If population P increase/decrease $r1$% first year, $r2$% increase/decrease second year and $r3$% increase/decrease third year, then after 3 years population =

$P(1 \pm \frac{r_1}{100})(1 \pm \frac{r_2}{100})(1 \pm \frac{r_3}{100})$

If Increase we use(+), if decrease we use (−)

6. If a man spend x% of this income on food, y% of remaining on rent and z% of remaining on cloths. If he has ₹ P remaining, then total income of man is = $\frac{P \times 100 \times 100 \times 100}{(100-x)(100-y)(100-z)}$

[Note: We can use this concept for area increase/decrease in mensuration for rectangle, triangle and parallelogram].

7. If P = Principal, R = Rate per annum, T = Time in years, SI = Simple interest, A = Amount

(a) SI= $\frac{PRT}{100}$

(b) A=P + SI=P[1+ $\frac{RT}{100}$]

8. If P = Principal, A = Amount in n years, R = rate of interest per annum.

$A=P[1+ \frac{RT}{100}]^n$, interest payable annually

9.

(i) $A=P[1+ \frac{R'}{100}]^{nT'}$,interest payable half−yearly $R' = \frac{R}{2}$, $n' = 2n$

(ii) $A=P[1+ \frac{RT}{100}]^{4n}$, interest payable quarterly;

10.

(i) [1+R/100] is the yearly growth factor;

(ii) [1+ $\frac{R}{100}$] is the yearly decay factor or depreciation factor.

11. Whentimeisfractionofayear,say $4\frac{3}{4}$, years,then

Amoutn=$P[1+ \frac{R}{100}]^4 \times [1+ \frac{\frac{3}{4}R}{100}]$

CI = Amount− Principal = $P[(1+ \frac{R}{100})^n - 1]$

WhenRatesaredifferentfordifferentyears,say R_1, R_2, R_3% for 1st, 2nd, 3rd yearsrespectively,then,

12. Amount=$P[1+ \frac{R_1}{100}][1+ \frac{R_2}{100}][1+ \frac{R_3}{100}]$

In general, interest is considered to be SIMPLE unless otherwise state

13. Present Value Under SI

The principal P is amounting to X in n periods. From this we know that

$X = P(1+ \frac{nr}{100}) \Rightarrow P = \frac{X}{(1+ \frac{nr}{100})}$

Hence, in general, the present value P of an amount X coming (or due) after n periods is given by

$P = \frac{X}{(1+ \frac{nr}{100})}$

where r is the rate percent per time period.

14. Present Value Under CI

The principal P is amounting to X in n periods. From this we know that

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Hence, in general, the present value P of an amount X coming (or due) after n periods is given by

$P = \frac{X}{P(1+ \frac{r}{100})^n}$

where r is the rate percentpertime period.

15. SAGR and CAGR

If sales increases fromxtoyinnyears, we can calculate the annualgrowthrateof sales in two ways, either using thesimpleinterest formula or using the compoundinterestformula. In this case the principal is consideredasxandthe amount is considered as y. ThustheSimpleAnnual Growth Rate(SAGR) is foundusing

$y - x = \frac{x \times r \times n}{100}$ and compoundAnnual Growth Rate (CAGR)is foundusing

$P(1+ \frac{r}{100})^n$ $y=x(1+ \frac{r}{100})^n$

$(CI)_2-(SI)_2= \frac{Pr^2}{100^2}$ for two years

$(CI)_3-(SI)_3= \frac{Pr^3}{100^3}+ \frac{3Pr^2}{100^2}$ for three years

16. If CP of x things = SP of y things, then

$\frac{Profit}{Loss} = [\frac{x-y}{y} \times 100]\%$

If +ve, Profit;

If −ve, Loss

17. If after selling x things P/L is equal to SP of y things, then P/L = $\frac{y}{(x \mp y)} \times 100$

[Profit = −
Loss= +]

18. If CP of two articles are same, and they sold at

Ist	IInd	overall
(x%) Profit	(y%) Profit	$(\frac{x+y}{2})\%$ Profit
(x%) Profit	(y%) Loss	$(\frac{x-y}{2})\%$ { Profit,if x>y; Loss,if x<y }
(x%) Loss	(y%) Loss	$(\frac{x+y}{2})\%$ Loss
(x%) Profit	(x%) Loss	No. profit, no Loss

19. If SP of two articles are same and they sold at

Ist	IInd	overall
Profit (x%)	Loss (x%)	Loss$(\frac{x^2}{100})\%$
(Profit (x%)	Loss (y%)	$(\frac{100(x-y)-2xy}{200+x-y})\%$ or $[\frac{2(100+x)(100-y)}{200+x-y}-100]\%$ { If+ve,then Profit%; If−ve,then Loss% }

20. After D% discount, requires P% profit, then total increase in C.P.=$[\frac{P+D}{100-D}\times 100]\%$

21. M.P.= C.P × $\frac{(100+P)}{(100-D)}$

22. Profit % = $\frac{(M.P.-C.P.)\times 100}{C.P.}$

23.

(i) For discount r1% and r2%, successive discount =

$$[(\frac{100+r_1}{100})(\frac{100+r_2}{100})(\frac{100+r_3}{100})-1]\times 100$$

(ii) For discount r1%, r2% and r2%, successive discount

$$[(\frac{100+r_1}{100})(\frac{100+r_2}{100})(\frac{100+r_3}{100})-1]\times 100$$

24. The Multiplying Factor because of cheating on volume = $\frac{Amount\ charged\ for\ (Reading)}{Actual\ Sold}$

TIME AND DISTANCE

1.

(i) If a body travels from point A to point B with a speed of p and back to point A (from point B) with a speed of q, then the average speed of the body can be calculated as 2pq/(p + q). Please note that this does not depend on the distance between A and B.

(ii) If a body covers part of the journey at speed p and the remaining part of the journey at speed q and the distances of the two parts of the journey are in the ratio m:n, then the average speed for the entire journey is (m + n)pq/(mq + np).

2. If u is the speed of the boat down the stream and v is the speed of the boat up the stream, then we have the following two relationships.

Speed of the boat in still water = (u + v)/2

Speed of the water current = (u – v)/2

3.

(i) If t_1 and t_2 time taken to travel from A to B and B to A, with speed a km/h and b km/h, then distance from A to B is

$d=(t_1+t_2)(\frac{ab}{a+b})$ $\quad$ $d=(t_1-t_2)(\frac{ab}{a-b})$

$d=(a-b)(\frac{t_1t_2}{t_1+t_2})$

(ii) If Ist part of distance is covered at the speed of a in t_1 time and the second part is covered at the speed of b in t_2 time, then the average speed = $(\frac{at_2+bt_1}{t_1+t_2})$

4. If A travels distance a and B travels distance b till their meeting, since they started simultaneously, we have $\frac{s_a}{s_b}=\frac{a}{b}$ where sa and sb are speed of A and B respectively.

Also after meeting A and B will have to travel distances b and a respectively to reach their ends.

Thus we have $t_a=\frac{b}{s_a}$ and $t_b=\frac{a}{s_b}$ i.e.

b = ta × sa and a = tb × sb.

Thus we have $\frac{a}{b}=\frac{t_b\times s_b}{t_a\times s_a}$.

Substituting in earlier equation we have

$\frac{sa}{sb} = \frac{b \times sb}{ta \times sa}$ given us the relation $\frac{sa}{sb} = \sqrt{\frac{tb}{ta}}$.

Please note this relation is to be used only when the two persons start simultaneously and the time given is the time taken after meeting to reach respective ends.

5.

(i) When TWO people are running around a circular track

Let the two people A and B with respective speeds of a and b (a > b) be running around a circular track (of length L) starting at the same point and at the same time. Then

	When the two persons are in the SAME direction	When the two persons are Running in OPPOSITE directions
Time taken to meet for the FIRST TIME EVER	$\frac{L}{(a-b)}$	$\frac{L}{(a+b)}$
Time taken to meet for the first time at the STARTING POINT	LCM of $\{\frac{L}{a}, \frac{L}{b}\}$	LCM of $\{\frac{L}{a}, \frac{L}{b}\}$

(ii) When THREE people are running around a circular track

Let the three people A, B and C with respective speeds of a, b and c (a > b > c) be running around a circular track (of length L) starting at the same point at the same time. In this case we consider the three persons running in the same direction as the general case.

Time taken to meet for the FIRST TIME EVER	LCM of $\{\frac{L}{(a-b)}, \frac{L}{(b-c)}\}$
Time taken to meet for the first time at the STARTING POINT	LCM of $\{\frac{L}{a}, \frac{L}{b}, \frac{L}{c}\}$

CLOCK

The hour hand moves $(\frac{1}{2})^o$ per minute, whereas the minute hand moves 6° per minute. The minute hand is constantly chasing the hour hand. The relative speed of the minute hand with respect to the hour hand is $5\frac{1}{2}^o$ per minute.

Difference of $5\frac{1}{2}^o$ comes in 1 min. So the difference of 360° will come in $\frac{360}{5\frac{1}{2}}$ min = $\frac{720}{11}$ min = $65\frac{5}{11}$ mm, and in a day,

[∵ $\frac{\text{Total minutes in a day}}{\text{Time (in minutes for one coincidence)}}$

$= \frac{24 \times 60}{\frac{720}{11}}$ times]

In a period of 12 hours, the hands make an angle of

- 0° with each other (i.e., they coincide with each other) 11 times and hence the time gap between two successive coincidences is 12/11 hours, i.e., $1\frac{1}{11}$ hours, i.e., $65\frac{5}{11}$ minutes.
- 180° with each other (i.e., they lie on the same straight line) 11 times.
- 90° or any other angle with each other 22 times.

TIME AND WORK

(i) If a man can do a piece of work in 20 days he will do $(\frac{1}{20})^{th}$ of the work in 1 day; and conversely if a man can do $(\frac{1}{20})^{th}$ of the work in 1 day, he will finish the work in 20 days.

(ii) If the number of men engaged to do a piece of work be changed in the ratio 5 : 4 the time required for the work will be changed in the ratio 4 : 5.

(iii) If A is thrice as good a workman as B, then A will take one-third of the time that B takes to do a certain work. OR Ratio of work done in same time 3 : 1

(iv) If A can do a work in a days, in one day = $\frac{1}{a}$ of total work

B can do it in b days, in one day = $\frac{1}{b}$ of total work

Together, they will finish $\frac{1}{a} + \frac{1}{b}$

(v) A can do a/b part of work in $t1$ days and c/d part of work in t_2 days, then $\frac{t_1}{a/b} = \frac{t_2}{c/d}$

(vi)

(a) If A is K times efficient than B, Then T(K + 1) = KtB

(b) If A is K times efficient than B and takes t days less than B

Then $T(K+1) = \frac{Kt}{K-1}$

KtB

$tB = \frac{Kt}{K-1}$

$\frac{t}{K-1}$

(vii) if 'm' men can do a piece of work in 'n' days & 'p' men can do the same work in 'q' days then: $m \times n = p \times q$

a) If Pipe Pa fills a tank of volume V ltrs in Ta hrs, and pipe P fills it in bT hrs.

Rate of filling; Pa $a = \frac{V}{Ta}$; Pb $b = \frac{V}{Tb}$ ltrs per hour

If together opened

Total water coming in (a + b) ltrs per hour

Time taken T $\frac{V}{(a+b)}$ hrs

OR $\frac{1}{T} = \frac{1}{Ta} + \frac{1}{Tb}$

b) If a cistern takes X min to be filled by a pipe but due to a leak, it takes Y extra minutes to be filled, then the time taken by leak to empty the cistern=($\frac{X2+XY}{Y}$) min

c) If a leak empty a cistern in X hours. A pipe which admits Y liters per hour water into the cistern and now cistern is emptied in Z hours, then capacity of cistern is=($\frac{X+Y+Z}{Z-Y}$) liters.

d) If two pipes A and B fill a cistern in x hours and y hours. A pipe is also an outlet C. If all the three pipes are opened together, the tank full in T hours. Then the time taken by C to empty the full tank is = $[\frac{xyT}{yT+xT-xy}]$

NUMBER SYSTEM

Remember!

Odd + Odd = Even

Even + Even = Even

Odd + Even = Odd

(Odd)Even = Odd

(Even)Odd = Even

Even × Even = Even

Odd × Odd = Odd

(Odd)Even ×(Even)Odd = Even

(Odd)Even +(Even)Odd = Odd

Prime Number :

Find the approx square root of given no. Divide the given no. by the prime no. less than approx square root of no. If given no. is not divisible by any of these prime no. then the no. is prime otherwise not.

For example : To check 359 is a prime number or not.

Sol. Approx sq. root = 19

Prime no. < 19 are 2, 3, 5, 7, 11, 13, 17

359 is not divisible by any of these prime nos. So 359 is a prime no. For example: Is 25001 + 1 is prime or not?

$\frac{25001 + 1}{2 + 1}$ ⇒ Reminder = 0,

∴ 25001 + 1 is not prime.

(i) There are 15 prime no. from 1 to 50.

(ii) There are 25 prime no. from 1 to 100.

(iii) There are 168 prime no. from 1 to 1000.

1. If a no. is in the form of xn + an, then it is divisible by (x + a); if n is odd.

2. If xn ÷ (x – 1), then remainder is always 1.

3. If xn ÷ (x + 1)

(i) If n is even, then remainder is 1.

(ii) If n is odd, then remainder is x.

4. Number of divisors :

(i) If N is any no. and N = an × bm × cp × … where a, b, c are prime no.

No. of divisors of N = (n + 1) (m + 1) (p + 1) ....

e.g. Find the no. of divisors of 90000.

N = 90000 = 22 × 32 × 52 × 102 = 22 × 32 × 52 × (2 × 5)2 = 24 × 32 × 54

So, the no. of divisors = (4 + 1) (2 + 1) (4 + 1) = 75

(ii) N=an× bm × cp, where a, b, c are prime

Then set of co-prime factors of N = [(n + 1) (m + 1) (p + 1) – 1 + nm + mp + pn + 3mnp]

(iii) If N = an × bm × cp..., where a, b & c are prime no. Then sum of the divisors $\frac{(an+1)-1)(bm+1)-1)(cP+1-1)}{(a-1)(b-1)(c-1)}$

5. To find the last digit or digit at the unit's place of an.

(i) If the last digit or digit at the unit's place of a is 1, 5 or 6, whatever be the value of n, it will have the same digit at unit's place, i.e.,

(.....1)n = (……1)

(.....5)n = (……5)

(.....6)n = (……6)

(ii) If the last digit or digit at the units place of a is 2, 3, 5, 7 or 8, then the last digit of an depends upon the value of n and follows a repeating pattern in terms of 4 as given below :

n	last digit of (…2)n	last digit of (…3)n	last digit of (…7)n	last digit of (…8)n
4x+1	2	3	7	8
4x+2	4	9	9	4
4x+3	8	7	3	2
4x	6	1	1	6

(iii) If the last digit or digit at the unit's place of a is either 4 or 9, then the last digit of an depends upon the value of n and follows repeating pattern in terms of 2 as given below.

n	last digit of (…4)n	last digit of (…9)n
2x	6	1
2x + 1	4	9

Some articles related to Divisibility :

(i) A no. of 3-digits which is formed by repeating a digit 3-times, then this no. is divisible by 3 and 37. e.g., 111, 222, 333, ...

(ii) A no. of 6-digit which is formed by repeating a digit 6-times then this no. is divisible by 3, 7, 11, 13 and 37. *e.g.*, 111111, 222222, 333333, 444444, ...

6. Divisible by 7: We use osculator (– 2) for divisibility test.

99995 : 9999 – 2 × 5 = 9989
9989 : 998 – 2 × 9 = 980
980 : 98 – 2 × 0 = 98
Now 98 is divisible by 7, so 99995 is also divisible by 7.

7. Divisible by 11: In a number, if difference of sum of digit at even places and sum of digit at odd places is either 0 or multiple of 11, then no. is divisible by 11.

For example, 12342 ÷ 11
Sum of even place digit = 2 + 4 = 6
Sum of odd place digit = 1 + 3 + 2 = 6
Difference = 6 – 6 = 0
∴ 12342 is divisible by 11.

8. Divisible by 13 : We use (+ 4) as osculator.

e.g., 876538 ÷ 13
876538: 8 × 4 + 3 = 35
5 × 4 + 3 + 5 = 28
8 × 4 + 2 + 6 = 40
0 × 4 + 4 + 7 = 11
1 × 4 + 1 + 8 = 13
13 is divisible by 13.
∴ 876538 is also divisible by 13.

9. Divisible by 17 : We use (– 5) as osculator.

e.g., 294678: 29467 – 5 × 8 = 29427
27427: 2942 – 5 × 7 = 2907
2907: 290 – 5 × 7 = 255
255: 25 – 5 × 5 = 0
∴ 294678 is completely divisible by 17.

10. Divisible by 19 : We use (+ 2) as osculator.

e.g: 149264: 4 × 2 + 6 = 14
4 × 2 + 1 + 2 = 11
1 × 2 + 1 + 9 = 12
2 × 2 + 1 + 4 = 9
9 × 2 + 1 = 19
19 is divisible by 19
∴ 149264 is divisible by 19.

11. HCF (Highest Common factor)

There are two methods to find the HCF–
(a) Factor method
(b) Division method

(i) For two no. a and b if a < b, then HCF of a and b is always less than or equal to a .

(ii) The greatest number by which x, y and z completely divisible is the HCF of x, y and z.

(iii) The greatest number by which x, y, z divisible and gives the remainder a, b and c is the HCF of (x –a), (y–b) and (z–c).

(vi) The greatest number by which x, y and z divisible and gives same remainder in each case, that number is HCF of (x–y), (y–z) and (z–x).

(v) H.C.F. of ,$\frac{a}{b}$ $\frac{c}{d}$ and $\frac{e}{f}$ = $\frac{H.C.M\ of\ (a,c,e)}{L.C.M.of\ (b,d,f)}$

12. LCM (Least Common Multiple)

There are two methods to find the LCM–
(a) Factor method
(b) Division method

(i) For two numbers a and b if a < b, then L.C.M. of a and b is more than or equal to b.

(ii) If ratio between two numbers is a : b and their H.C.F. is x, then their L.C.M. = abx.

(iii) If ratio between two numbers is a : b and their L.C.M. is x, then their H.C.F. = $\frac{x}{ab}$

(vi) The smallest number which is divisible by x, y and z is L.C.M. of x, y and z.

(v) The smallest number which is divided by x, y and z give remainder a, b and c, but (x – a) = (y – b) = (z – c) = k, then number is (L.C.M. of (x, y and z) – k).

(vi) The smallest number which is divided by x, y and z give remainder k in each case, then number is (L.C.M. of x, y and z) + k.

(vii) L.C.M. of $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f} = \frac{L.C.M\ of\ (a,c,e)}{H.C.M.of\ (b,d,f)}$

(viii) For two numbers a and b –
LCM × HCF = a × b

(ix) If a is the H.C.F. of each pair from n numbers and L is L.C.M., then product of n numbers = $a^{n-1}.L$

Properties of HCF AND LCM

- The HCF of two or more numbers is smaller than or equal to the smallest of those numbers.
- The LCM of two or more numbers is greater than or equal to the largest of those numbers
- If numbers N1, N2, N3, N4 etc. give remainders R1, R2, R3, R4, respectively, when divided by the same number P, then P is the HCF of (N1 – R1), (N2 – R2), (N3 – R3), (N4 – R4) etc.
- If the HCF of numbers N1, N2, N3 ... is H, then N1, N2, N3... can be written as multiples of H (Hx, Hy, Hz..) Since the HCF divides all the numbers, every number will be a multiple of the HCF.
- If the HCF of two numbers N1 and N2 is H, then, the numbers (N1 + N2) and (N1 – N2) are also divisible by H. Let N1 = Hx and N2 = Hy, since the numbers will be multiples of H. Then, N1+N2 = Hx + Hy = H(x + y), and N1 – N2 = Hx – Hy = H(x – y). Hence both the sum and differences of the two numbers are divisible by the HCF.
- If numbers N1, N2, N3, N4 etc. give an equal remainder when divided by the same number P, then P is a factor of (N1 – N2), (N2 – N3), (N3 – N4)...
- If L is the LCM of N1, N2, N3, N4.. all the multiples of L are divisible by these numbers.
- If a number P always leaves a remainder R when divided by the numbers N1, N2, N3, N4 etc., then P = LCM (or a multiple of LCM) of N1, N2, N3, N4.. + R.

1. Highest power of prime number p in n! = $[\frac{n}{p}] + [\frac{n}{p^2}] + [\frac{n}{p^3}] + [\frac{n}{p^4}]$ + ...where [x] denotes the greatest integer less than or equal to x.

2. **Number of numbers less than or prime to a given number:**
If N is a natural number such that N = $a^p \times b^q \times c^r$, where a, b, c are different prime factors and p, q, r are positive integers, then the number of positive integers less than and prime to N =
$N(1-\frac{1}{a})(1-\frac{1}{b})(1-\frac{1}{c})$. Therefore, $N(1-\frac{1}{a})(1-\frac{1}{b})(1-\frac{1}{c})$ number have **no** factor in common with N.

3. **Remainder Concept**
Suppose the numbers N1, N2, N3… give quotients Q1, Q2, Q3… and remainders R1, R2, R3..., respectively, when divided by a common divisor D.
Therefore N1 = D × Q1 + R1,
N2 = D × Q2 + R2,
N3 = D × Q3 + R3.. and so on.
Let P be the product of N1, N2, N3…
Therefore, P = N1N2N3.. = (D × Q1 + R1)(D × Q2 + R2)(D × Q3 + R3)..
= D × K + R1R2R3... where K is some number ….(1)
→ In the above equation, since only the product R1R2R3… is free of D, therefore the remainder when P is divided by D is the remainder when the product R1R2R3… is divided by D.
Let S be the sum of N1, N2, N3…
Therefore, S = (N1) + (N2) + (N3) +... = (D × Q1 + R1) + (D × Q2 + R2) + (D × Q3 + R3).
= D × K + R1 + R2 + R3… where K is some number---(2)
→ Hence the remainder when S is divided by D is the remainder when R1 + R2 + R3 is divided by D.

4. **Some special cases:**
Euler's Theorem
If M and N are two numbers co-prime to each other, i.e. HCF(M, N) = 1 and N = $a^p b^q c^r$..., Remainder $[\frac{M^{\phi(N)}}{N}]$ = 1, where $\phi(N) = N(1-\frac{1}{a})(1-\frac{1}{b})(1-\frac{1}{c})$...and is known as Euler's Totient function. ϕ(N) is also the number of numbers less than and prime to N.

Wilson's Theorem: If p is prime, (p – 1)! + 1 is a multiple of p.
Fermat's theorem: If p is prime and HCF (a, p) = 1, then $a^{p-1} - 1$ is divisible by p.

5.
(i) $x^n - y^n$ is divisible by (x + y)
When n is even
(ii) $x^n - y^n$ is divisible by (x – y) When n is either odd or even.

6. For any integer n, n3 – n is divisible by 3, n5 – n is divisible by 5, n11 – n is divisible by 11, n13 – n is divisible by 13.

PROGRESSIONS

(i) Sum of n natural number = $\frac{(n)(n+1)}{2}$

(ii) Sum of n even number = (n) (n + 1)

(iii) Sum of n odd number = n2

(iv) Sum of sq. of first n natural no. = $\frac{n(n+1)(2n+1)}{6}$

(v) Sum of sq. of first n odd natural no. = $\frac{n(4n^2-1)}{3}$

(vi) Sum of sq. of first n even natural no. = $\frac{2n(n+1)(2n+1)}{3}$

(vii) Sum of cube of first n natural no. = $\frac{n^2(n+1)^2}{4}$ = $[\frac{n(n+1)}{2}]^2$

(viii) Sum of cube of first n even natural no. = $2n^2(n+1)^2$

(ix) Sum of cube of first n odd natural no. = $n^2(2n^2-1)$

General term of an A.P.

i. Let 'a' be the first term and 'd' be the common difference of an A.P. Then its nth term is $a+(n-1)d$

$i.e. T_n = a+(n-1)d$

ii. ***p*th term of an A.P. from the end :** Let 'a' be the first term and 'd' be the common difference of an A.P. having n terms. Then pth term from the end is $(n-p+1)th$ term from the beginning

$i.e.$, pth term from the end $= T_{(n-p+1)} = a+(n-p)d.$

If last term of an A.P. is l then pth term from end $= l-(p-1)d$

Sum of *n* terms of an A.P

The sum of n terms of the series $a+(a+d)+(a+2d)+\ldots+(a+(n-1)d)$ is given by

$S_n = \frac{n}{2}[2a+(n-1)d]$

Also, $S_n = \frac{n}{2}(a+l)$, where l = last term = $a+(n-1)d$.

Note: If a, A, b are in A.P., then A is called A.M. between a and b.

(1) If $a, A_1, A_2, A_3, \ldots, A_n, b$ are in A.P., then $A_1, A_2, A_3, \ldots, A_n$ are called n A.M.'s between a and b.

General term of a G.P.

(1) We know that, $a, ar, ar^2, ar^3, \ldots, ar^{n-1}$ is a sequence of G.P.

Here, the first term is 'a' and the common ratio is 'r'.

The general term or nth term of a G.P. is $T_n = ar^{n-1}$.

It should be noted that, $r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \ldots$

(2) ***p*th term from the end of a finite G.P. :** If G.P. consists of 'n' terms, pth term from the end $= (n-p+1)th$ term from the beginning $= ar^{n-p}$.

Also, the pth term from the end of a G.P. with last term l and common ratio r is $l\left(\frac{1}{r}\right)^{n-1}$.

Sum of first '*n*' terms of a G.P.

If a be the first term, r the common ratio, then sum S_n of first n terms of a G.P. is given by

$S_n = \frac{a(1-r^n)}{1-r}$ and $S_n = \frac{a-lr}{1-r}$, (when $|r|< 1$)

$S_n = \frac{a(r^n-1)}{r-1}$ and $S_n = \frac{lr-a}{r-1}$, (when $|r|> 1$)

$S_n = na$, (when $r = 1$)

Sum of infinite terms of a G.P.

When $|r|< 1$, (or $-1<r<1$); $S_\infty = \frac{a}{1-r}$

INDICES SURDS AND LOGARITHMS

Logarithms

Let m and n be arbitrary positive numbers such that $a>0, a\neq 1, b>0, b\neq 1$ then

(1) $\log_a a = 1, \log_a 1 = 0$

(2) $\log_a b . \log_b a = 1 \Rightarrow \log_a b = \frac{1}{\log_b a}$

(3) $\log_c a = \log_b a . \log_c b$ or $\log_c a = \frac{\log_b a}{\log_b c}$

(4) $\log_a (mn) = \log_a m + \log_a n$

(5) $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

(6) $\log_a m^n = n\log_a m$

(7) $a^{\log_a m} = m$

(8) $\log_a \left(\frac{1}{n}\right) = -\log_a n$

(9) $\log_{a^\beta} n = \frac{1}{\beta}\log_a n$

(10) $\log_{a^\beta} n^\alpha = \frac{\alpha}{\beta}\log_a n, (\beta\neq 0)$

(11) $a^{\log_c b} = b^{\log_c a}, (a,b,c>0$ and $c\neq 1)$

Indices

(1) $a^0 = 1$, (a ≠ 0)

(2) $a^{-m} = \frac{1}{a^m}$, (a ≠ 0)

(3) $a^m . a^n = a^{m+n}$, where m and n are rational numbers

(4) $a^{m-n} = \frac{a^m}{a^n}$, where m and n are rational numbers, $a\neq 0$

(5) $(a^m)^n = a^{mn}$

(6) $a^{\frac{p}{q}} = \sqrt[q]{a^p}$

(7) If $x = y$, then $a^x = a^y$, but the converse may not be true.

Surds

The following rules of radicals are useful to simplify surds:

(1) $[\sqrt[n]{a}]^n = a$

(2) $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$

(3) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

RATIONALIZING FACTOR OF A SUM OF SURDS

Consider the term $\sqrt{a}+\sqrt{b}$ which is the sum of two surds. The rationalizing factor for this is $\sqrt{a}-\sqrt{b}$, When we multiply these two terms we get,

$(\sqrt{a}+\sqrt{b})\times(\sqrt{a}-\sqrt{b}) = a-b$

This brings us to the concept of the conjugate. The conjugate of $(\sqrt{a}+\sqrt{b})$ is $(\sqrt{a}-\sqrt{b})$. Similarly, the conjugate of $(\sqrt{a}-\sqrt{b})$ is $(\sqrt{a}+\sqrt{b})$. The conjugate is the rationalizing factor (RF) for a sum of surds of the form $(\sqrt{a} \pm \sqrt{b})$.

(i) Value of $\sqrt{P+\sqrt{P+\sqrt{P+\ldots\ldots\infty}}} = \frac{\sqrt{4P+1}+1}{2}$

(ii) Value of $\sqrt{P-\sqrt{P-\sqrt{P-\ldots\ldots\infty}}} = \frac{\sqrt{4P+1}-1}{2}$

(iii) Value of $\sqrt{P.\sqrt{P.\sqrt{P\ldots\ldots\infty}}} = P$

(iv) Value of $\sqrt{P\sqrt{P\sqrt{P\sqrt{P\sqrt{P}}}}} = P^{(2^n-1)\div 2^n}$

[Where n → no. of times P repeated].

Note: If factors of P are n & (n + 1) type then value of $\sqrt{P+\sqrt{P+\sqrt{P+\ldots\infty}}} = (n+1)$ and $\sqrt{P-\sqrt{P-\sqrt{P-\ldots\infty}}} = n$

QUADRATIC EQUATION AND SPECIAL EQUATION

GENERAL EQUATION OF Nth DEGREE

Let polynomial $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n$.
where $a_0, a_1, a_2, \ldots a_n$ are rational numbers and $n>0$.
Then the values of x for which f(x) reduces to zero are called root of the equation f(x)=0. The highest whole number power of x is called the degree of the equation.

For example

$x^4-3x^3+4x^2+x+1=0$ is an equation with degree four.

$x^5-6x^4+3x^2+1=0$ is an equation with degree five.

$ax+b=0$ is called the linear equation.

$ax^2+bx+c=0$ is called the quadratic equation.

$ax^3+bx^2+cx+d=0$ is called the cubic equation.

Higher degree equations

The equation $p(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n = 0 \ldots\ldots$(i)

Where the coefficients $a_0, a_1, \ldots\ldots, a_n \in R$ (or C) and $a_0 \neq 0$ is called an equation of n^{th} degree, which has exactly n roots $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n \in C$, then we can write

$p(x) = a_0(x-\alpha_1)(x-\alpha_2)\ldots\ldots(x-\alpha_n)$

$= a_0\{x^n - (\Sigma\alpha_1)x^{n-1} + (\Sigma\alpha_1\alpha_2)x^{n-2} - \cdots + (-1)^n\alpha_1\alpha_2\ldots\alpha_n\}$ ……(ii)

Comparing (i) and (ii),

$\Sigma\alpha_1 = \alpha_1+\alpha_2+\cdots+\alpha_n = -\frac{a_1}{a_0}$

$\Sigma\alpha_1\alpha_2 = \alpha_1\alpha_2+\cdots+\alpha_{n-1}\alpha_n = \frac{a_2}{a_0}$

and so on and $\alpha_1\alpha_2\ldots\alpha_n = (-1)^n\frac{a_n}{a_0}$

Cubic equation: When $n=3$, the equation is a cubic of the form $ax^3+bx^2+cx+d=0$, and we have in this case

$\alpha+\beta+\gamma = -\frac{b}{a}$; $\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a}$; $\alpha\beta\gamma = -\frac{d}{a}$

Biquadratic equation: If $\alpha, \beta, \gamma, \delta$ are roots of the biquadratic equation $ax^4+bx^3+cx^2+dx+e=0$, then

$\Sigma\alpha_1 = \alpha+\beta+\gamma+\delta = -\frac{b}{a}$

$\sigma_2 = \frac{c}{a}$

$\sigma_3 = -\frac{d}{a}$

$\sigma_4 = \frac{e}{a}$.

Properties of quadratic equation

(a) If $f(a)$ and $f(b)$ are of opposite signs then at least one or in general odd number of roots of the equation $f(x) = 0$ lie between a and b.

(b) If $f(a) = f(b)$ then there exists a point c between a and b such that $f'(c) = 0$, $a < c < b$.

(c) If α is a root of the equation $f(x) = 0$ then the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$, then $(x - \alpha)$ is factor of $f(x)$.

(d) If the roots of the quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ are in the same ratio

i.e. $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$ then $b_1^2/b_2^2 = a_1c_1 / a_2c_2$.

The quadratic expression

(a) Let $f(x) = ax^2 + bx + c$, $a,b,c \in R$, $a \neq 0$ be a quadratic expression.

Since, $f(x) = a\left\{\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2}\right\}$ ...(i)

The following is true from equation (i)

(i) $f(x) > 0 \; (< 0)$ for all values of $x \in R$ if and only if $a > 0 \; (< 0)$ and $D \equiv b^2 - 4ac < 0$.

(ii) $f(x) \geq 0 \; (\leq 0)$ if and only if $a > 0 \; (< 0)$ and $D = b^2 - 4ac = 0$.

In this case $(D = 0)$, $f(x) = 0$ if and only if $x = -\frac{b}{2a}$

(iii) If $D = b^2 - 4ac > 0$ and $a > 0 \; (<0)$, then

$f(x)$ <0 (>0), for x lying between the roots of f(x)=0

>0 (<0), for x not lying between the roots of f(x)=0

{=0, for x = each of the roots of f(x)=0

(iv) If $a > 0, (< 0)$, then $f(x)$ has a minimum (maximum) value at $x = -\frac{b}{2a}$ and this value is given by

$$[f(x)]_{\min(\max)} = \frac{4ac - b^2}{4a}.$$

(b) **Sign of quadratic expression :** Let $f(x) = ax^2 + bx + c$ or $y = ax^2 + bx + c$

Where $a, b, c \in R$ and $a \neq 0$, for some values of x, $f(x)$ may be positive, negative or zero. This gives the following cases :

(i) $a > 0$ and $D < 0$, so $f(x) > 0$ for all $x \in R$ *i.e.*, $f(x)$ is positive for all real values of x.

(ii) $a < 0$ and $D < 0$, so $f(x) < 0$ for all $x \in R$ *i.e.*, $f(x)$ is negative for all real values of x.

(iii) $a > 0$ and $D = 0$, so $f(x) \geq 0$ for all $x \in R$ *i.e.*, $f(x)$ is positive for all real values of x except at vertex, where $f(x) = 0$.

(iv) $a < 0$ and $D = 0$, so $f(x) \leq 0$ for all $x \in R$ *i.e.* $f(x)$ is negative for all real values of x except at vertex, where $f(x) = 0$.

(v) $a > 0$ and $D > 0$, let $f(x) = 0$ have two real roots α and β $(\alpha < \beta)$, then $f(x) > 0$ for all $x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) < 0$ for all $x \in (\alpha, \beta)$.

(vi) $a < 0$ and $D > 0$, let $f(x) = 0$ have two real roots α and β $(\alpha < \beta)$. Then $f(x) < 0$ for all $x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) > 0$ for all $x \in (\alpha, \beta)$

(c) **Graph of a quadratic expression**

We have $y = ax^2 + bx + c = f(x)$

$y = a\left\{\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right\}$ $\Rightarrow$ $y + \frac{D}{4a} = a\left(x + \frac{b}{2a}\right)^2$

Now, let $y + \frac{D}{4a} = Y$ and $X = x + \frac{b}{2a}$

$Y = a.X^2 \Rightarrow X^2 = \frac{1}{a}Y$

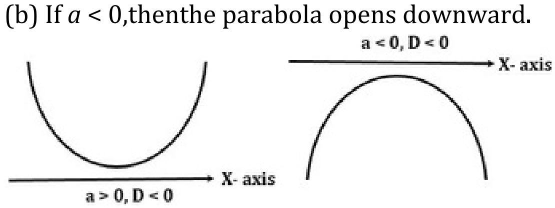
(i) The graph of the curve $y = f(x)$ is parabolic.

(ii) The axis of parabola is $X = 0$ or $x + \frac{b}{2a} = 0$ *i.e.* (parallel to y-axis).

(iii)

(a) If $a > 0$, then the parabola opens upward.

(b) If $a < 0$, then the parabola opens downward.



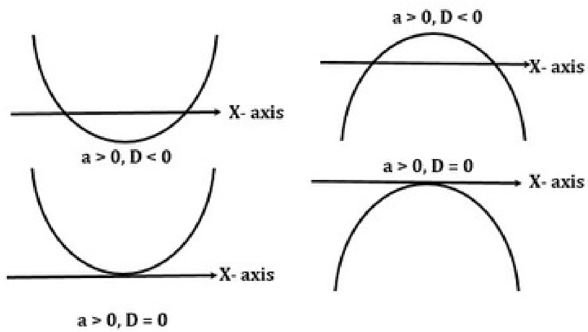
(iv) Intersection with axis

(a) Intersection with x-axis : For x axis, $y = 0$

$\Rightarrow ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{D}}{2a}$

For $D > 0$, parabola cuts x-axis in two real and distinct points *i.e.* $x = \frac{-b \pm \sqrt{D}}{2a}$.

For $D=0$, parabola touches x-axis in one point, $x = -b/2a$.



(b) **Intersection with axis y-axis :** For y axis $x = 0, y = c$.

Inequalities

If a and b are two positive and real numbers and a>b then

$a+c>b+c$, where c is a real number

$a-c>b-c$, where c is a real number

If $a>b$ and $c>d$, then $ac>bd$ but we cannot say that $ad>bc$.

$\frac{a}{c} > \frac{b}{c}$ if c is positive and real but $\frac{a}{c} < \frac{b}{c}$ if c is negative and real

$ac>bc$ if c is positive and real but $ac<bc$ if c is negative and real

$a^n>b^n$, where n is a positive number

$\frac{1}{a^n} < \frac{1}{b^n}$, where n is a positive number

Arithmetic mean ≥ geometric mean which means

$\frac{a+b}{2} \geq \sqrt{ab}$ and $\frac{a_1+a_2+a_3+\cdots+a_n}{n} \geq (a_1 a_2 \ldots a_n)^{\frac{1}{n}}$

FUNCTIONS AND GRAPHS

(1) **Absolute value:** The absolute value of a number x, denoted by $|x|$, is a number that satisfies the conditions

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ x & \text{if } x > 0 \end{cases}$$

We also define $|x|$ as follows,

$|x|$ = maximum $\{x, -x\}$ or $|x| = \sqrt{x^2}$.

(2) **Algebraic functions:** Functions consisting of finite number of terms involving powers and roots of the independent variable and the four fundamental operations +, −, × and ÷ are called algebraic functions. *e.g.,*

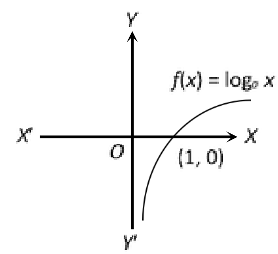
(i) $x^{\frac{3}{2}} + 5x$ (ii) $\frac{\sqrt{x}+1}{x-1}, x \neq 1$

(iii) $3x^4 - 5x + 7$

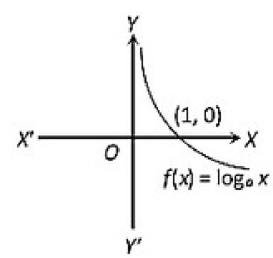
(3) **Transcendental function:** A function which is not algebraic is called a transcendental function. *e.g.*, trigonometric; inverse trigonometric, exponential and logarithmic functions are all transcendental functions.

(i) **Trigonometric functions :** A function is said to be a trigonometric function if it involves circular functions (sine, cosine, tangent, cotangent, secant, cosecant) of variable angles.

(ii) **Logarithmic function :** Let $a \neq 1$ be a positive real number. Then $f:(0, \infty) \to R$ defined by $f(x) = \log_a x$ is called logarithmic function. Its domain is $(0, \infty)$ and range is R.



Graph of $f(x) = \log_a x$, when $a > 1$



Graph of $f(x) = \log_a x$, when $a < 1$

(iii) **Exponential function :** Let $a \neq 1$ be a positive real number. Then $f:R \to (0, \infty)$ defined by $f(x) = a^x$ called exponential function. Its domain is R and range is $(0, \infty)$.

(4) **Explicit and implicit functions :** A function is said to be explicit if it can be expressed directly in terms of the independent variable. If the function can not be expressed directly in terms of the independent

variable or variables, then the function is said to be implicit. *e.g.* $y = \sin^{-1}x + \log x$ is explicit function, while $x^2 + y^2 = xy^3$ and $(a - x)^2 (b - y)^2$ are implicit functions.

(5) **Modulus function:** The function defined by $f(x) = |x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$ is called the modulus function.

(6) **Signum function:** The function defined by $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ or $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is called the signum function. The domain is R and the range is the set {–1, 0, 1}.

(7) **Even function**: If we put $(-x)$ in place of x in the given function and if $f(-x) = f(x)$, $\forall x \in$ domain then function $f(x)$ is called even function. *e.g.* $f(x) = e^x + e^{-x}$, $f(x) = x^2$, $f(x) = x\sin x$, $f(x) = \cos x$, $f(x) = x^2\cos x$ all are even functions.

(8) **Odd function:** If we put $(-x)$ in place of x in the given function and if $f(-x) = -f(x)$, $\forall x \in$ domain then $f(x)$ is called odd function. *e.g.*, $f(x) = e^x - e^{-x}$, $f(x) = \sin x$, $f(x) = x^3$, $f(x) = x\cos x$, $f(x) = x^2\sin x$ all are odd functions.

(9) **Greatest integer function:** Let $f(x) = [x]$, $[x]$ denotes the greatest integer less than or equal to x. The domain is R and the range is I. *e.g.* [1.1] = 1, [2.2] = 2, [– 0.9] = –1, [– 2.1] = – 3 etc. The function f defined by $f(x) = [x]$ for all $x \in R$, is called the greatest integer function.

(10) **Periodic function**: A function is said to be periodic function if its each value is repeated after a definite interval. So a function $f(x)$ will be periodic if a positive real number T exist such that, $f(x + T) = f(x)$, $\forall x \in$ domain. Here the least positive value of T is called the period of the function.

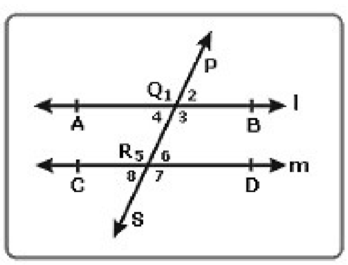
(11) Composite function: *If $f : A \to B$ and $g : B \to C$ are two function then the composite function of f and g, $gof\, A \to C$ will be defined as $gof(x) = g[f(x)], \forall x \in A$*

Properties of composition of function :

(i) f is even, g is even $\Rightarrow$ *fog* even function.
(ii) f is odd, g is odd $\Rightarrow$ *fog* is odd function.
(iii) *f* is even, g is odd $\Rightarrow$ *fog* is even function.
(iv) *f* is odd, g is even $\Rightarrow$ *fog* is even function.

GEOMETRY AND MENSURATION

Angles formed by a transversal of two parallel lines:



In the above figure, l and m are two parallel lines intersected by a transversal PS. The following properties of the angles can be observed:

∠3 = ∠5 and ∠4 = ∠6 **[Alternate angles]**

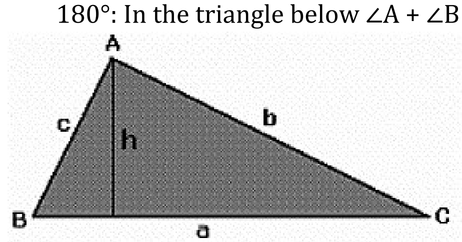
∠1 = ∠5, ∠2 = ∠6, ∠4 = ∠8, ∠3 = ∠7 **[Corresponding angles]**

∠4 + ∠5 = ∠3 + ∠6 = 180° **[Supplementary angles]**

General Properties of Triangles:

1. The sum of the two sides is greater than the third side: a + b > c, a + c > b, b + c > a

2. The sum of the three angles of a triangle is equal to 180°: In the triangle below ∠A + ∠B + ∠C = 180°



3. **Area of a Triangle:**

Area of a triangle = $\frac{1}{2} \times base \times height = \frac{1}{2} \times a \times h$

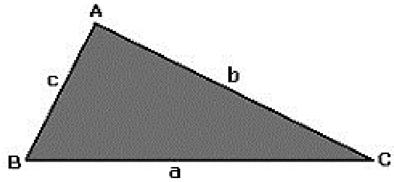
Area of a triangle = $\frac{1}{2} bc\sin A = \frac{1}{2} ab\sin C = \frac{1}{2} ac\sin B$

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

Area of a triangle = $\frac{abc}{4R}$ where R = circumradius

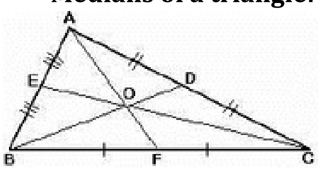
Area of a triangle = r × s where r = inradius and $s = \frac{a+b+c}{2}$

4. **More Rules:**



- Sine $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Cosine $\cos A = \frac{b2+c2-a2}{2bc}$, $\cos B = \frac{a2+c2-b2}{2ac}$, $\cos C = \frac{b2+a2-c2}{2ab}$

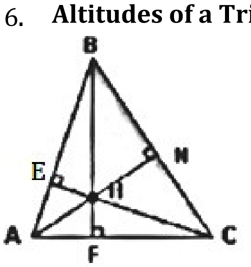
5. **Medians of a triangle:**



The medians of a triangle are lines joining a vertex to the midpoint of the opposite side. In the figure, AF, BD and CE are medians. The point where the three medians intersect is known as the centroid. O is the centroid in the figure.

- The medians divide the triangle into two equal areas. In the figure, area ΔABF = area ΔAFC = area ΔBDC = area ΔBDA = area ΔCBE = area ΔCEA = $\frac{\text{Area } \Delta ABC}{2}$
- The centroid divides a median internally in the ratio 2: 1. In the figure, $\frac{AO}{OF} = \frac{BO}{OD} = \frac{CO}{OE} = 2$
- Apollonius Theorem: AB2 + AC2 = 2(AF2 + BF2) or BC2 + BA2 = 2(BD2 + DC2) or BC2 + AC2 = 2(EC2 + AE2)

6. **Altitudes of a Triangle:**

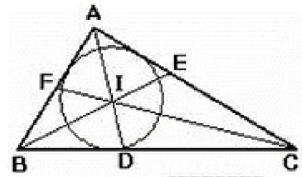


The altitudes are the perpendiculars dropped from a vertex to the opposite side. In the figure, AN, BF, and CE are the altitudes, and their point of intersection, H, is known as the orthocenter.

Triangle ACE is a right-angled triangle. Therefore, ∠ECA = 90° − ∠A. Similarly in triangle CAN, ∠CAN = 90° − ∠C. In triangle AHC, ∠CHA = 180° − (∠HAC + ∠HCA) = 180° − (90° − ∠A + 90° − ∠C) = ∠A + ∠C = 180° − ∠B.

Therefore, ∠AHC and ∠B are supplementary angles.

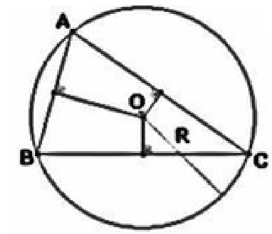
7. **Internal Angle Bisectors of a Triangle:**



In the figure above, AD, BE and CF are the internal angle bisectors of triangle ABC. The point of intersection of these angle bisectors, I, is known as the incentre of the triangle ABC, i.e. centre of the circle touching all the sides of a triangle.

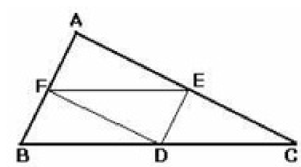
- ∠BIC = 180° − (∠IBC + ∠ICB) = $180 - (\frac{B}{2} + \frac{C}{2}) = 180 - (\frac{B+C}{2}) = 180 - (\frac{180-A}{2}) = 90 + \frac{A}{2}$
- $\frac{AB}{AC} = \frac{BD}{CD}$ (internal bisector theorem)

8. **Perpendicular Side Bisectors of a Triangle:**



In the figure above, the perpendicular bisectors of the sides AB, BC and CA of triangle ABC meet at O, the circumcentre (centre of the circle passing through the three vertices) of triangle ABC. In figure above, O is the centre of the circle and BC is a chord. Therefore, the angle subtended at the centre by BC will be twice the angle subtended anywhere else in the same segment. Therefore, ∠BOC = 2∠BAC.

9. **Line Joining the Midpoints:**



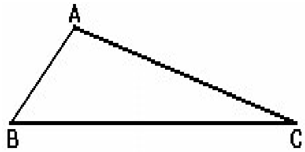
In the figure above, D, E and F are midpoints of the sides of triangle ABC. It can be proved that:

- FE // BC, DE // AB and DF // AC.
- FE= $\frac{BC}{2}$,DE= $\frac{AB}{2}$,FD= $\frac{AC}{2}$

  Area ΔDEF = Area ΔAFE = Area ΔBDF Area ΔDEC = $\frac{Area\ \Delta ABC}{4}$
- **Corollary:** If a line is parallel to the base and passes through midpoint of one side, it will pass through the midpoint of the other side also.

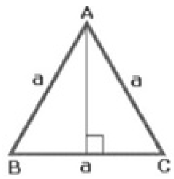
Types of triangles:

Scalene Triangle



No side equal.
All the general properties of triangle apply

Equilateral Triangle



Each side equal
Each angle = 60°

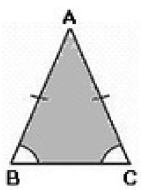
Length of altitude = $\frac{\sqrt{3}}{2}a$

Area = $\frac{\sqrt{3}}{4}a2$

Inradius = $\frac{a}{2\sqrt{3}}$

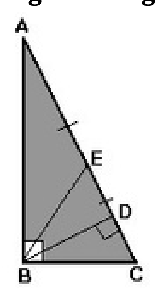
Circumradius = $\frac{a}{\sqrt{3}}$

Isosceles Triangle



Two sides equal.
The angles opposite to the opposite sides are equal.

Right Triangle



One of the angles is a right angle, i.e. 90°.

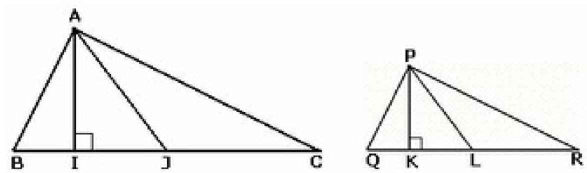
Area = $\frac{1}{2}AB\times BC$

AC2 = AB2 + BC2

Altitude BD = $\frac{AB\times BC}{AC}$

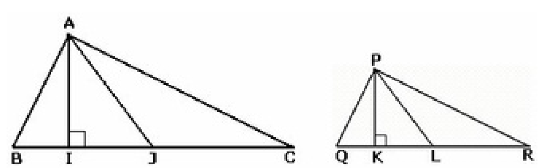
The midpoint of the hypotenuse is equidistant from all the three vertices, i.e. EA = EB = EC

Similarity of triangles:



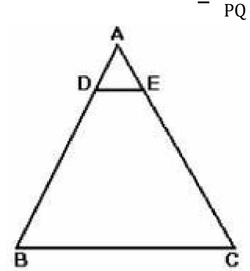
Two triangles are similar if their corresponding angles are equal or corresponding sides are in proportion.
In the figure given above, triangle ABC issimilar to triangle PQR. Then ∠A = ∠P, ∠B = ∠Q and ∠C= ∠R and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{AI}{PK}$ (altitudes) = $\frac{AJ}{PL}$ (medians)
Therefore, if you need to prove two triangles similar, prove their corresponding angles to be equal or their corresponding sides to be in proportion.

Ratio of Areas:

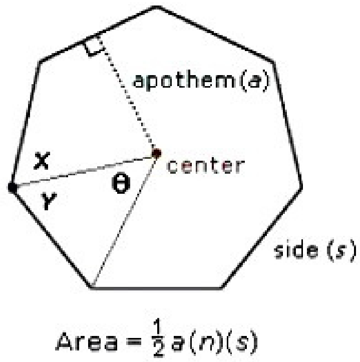


If two triangles are similar, the ratioof their areas is the ratio of the squares of the length oftheir corresponding sides. Therefore,

$$\frac{\text{Area of triangle ABC}}{\text{Area of triangle PQR}} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$



Regular polygon: A regular polygon is a polygon with all its sides equal and all its interior angles equal. All vertices of a regular lie on a circle whose center is the center of the polygon.



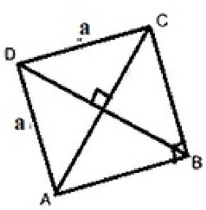
Each side of a regular polygon subtends an angle θ = $\frac{360}{n}$ at the centre, as shown in the figure.

Also = Y = $\frac{180-\frac{360}{n}}{2}$ = $\frac{180(n-2)}{2n}$. Therefore, interior angle of a regular polygon = $X+Y$ = $\frac{180(n-2)}{n}$.

Sum of all the angles of a regular polygon = $n\times \frac{180(n-2)}{n}$ = $180(n-2)$

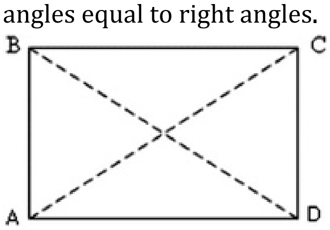
Quadrilateral: A quadrilateral is any closed shape that has four sides. The sum of the measures of the angles is 360o. Some of the known quadrilaterals are square, rectangle, trapezium, parallelogram and rhombus.

Square: A square is regular quadrilateral that has four right angles and parallel sides. The sides of a square meet at right angles. The diagonals also bisect each other perpendicularly.



If the side of the square is a, then its perimeter = 4a, area = a2 and the length of the diagonal = $\sqrt{2}a$

Rectangle: A rectangle is a parallelogram with all its angles equal to right angles.



Properties of a rectangle:

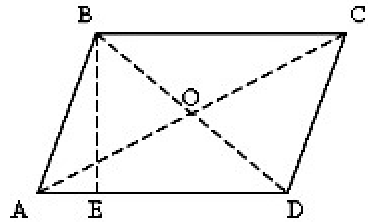
Sides of rectangle are its heights simultaneously.

Diagonals of a rectangle are equal: AC = BD.

A square of a diagonal length is equal to a sum of squares of its sides' lengths, i.e. $AC^2 = AD^2 + DC^2$.

Area of a rectangle = length × breadth

Parallelogram: A parallelogram is a quadrangle in which opposite sides are equal and parallel.



Any two opposite sides of a parallelogram are called bases, a distance between them is called a height.

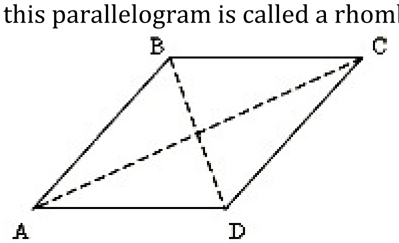
Area of a parallelogram = base × height

Perimeter = 2(sum of two consecutive sides)

Properties of a parallelogram:

1. Opposite sides of a parallelogram are equal (AB = CD, AD = BC).
2. Opposite angles of a parallelogram are equal (∠A=∠C, ∠B =∠D).
3. Diagonals of a parallelogram are divided in their intersection point into two (AO = OC, BO = OD).
4. A sum of squares of diagonals is equal to a sum of squares of four sides: $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$.

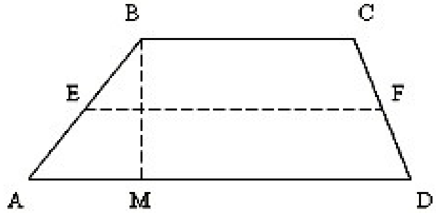
Rhombus: If all sides of parallelogram are equal, then this parallelogram is called a rhombus.



Diagonals of a rhombus are mutually perpendicular (AC ⊥ BD) and divide its angles into two (∠DCA = ∠BCA, ∠ABD = ∠CBD etc.).

Area of a rhombus = $\frac{1}{2}$ product of diagonals = $\frac{1}{2}$ × AC × BD

Trapezoid: Trapezoid is a quadrangle two opposite sides of which are parallel.



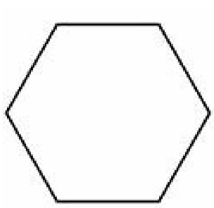
Here AD || BC. Parallel sides are called bases of a trapezoid, the two others (AB and CD) are called lateral sides. A distance between bases (BM) is a height. The segment EF, joining midpoints E and F of the lateral sides, is called a midline of a trapezoid. A midline of a trapezoid is equal to a half-sum of bases: $EF = \frac{AD+BC}{2}$ and parallel to them: EF || AD and EF || BC. A trapezoid with equal lateral sides (AB = CD) is called an isosceles trapezoid. In an isosceles trapezoid angles by each base, are equal (∠A = ∠D, ∠B = ∠C)..

Area of a trapezoid = $\frac{\text{Sum of parallel sides}}{2} \times height = \frac{AD+BC}{2} \times BM$

In a trapezium ABCD with bases $\overline{AB}$ and $\overline{CD}$, the sum of the squares of the lengths of the diagonals is equal to the sum of the squares of the lengths of the non-parallel sides and twice the product of the lengths of the parallel sides:AC2 + BD2 = AD2 + BC2 + 2·AB·CD

Here is one more polygon, a regular hexagon:

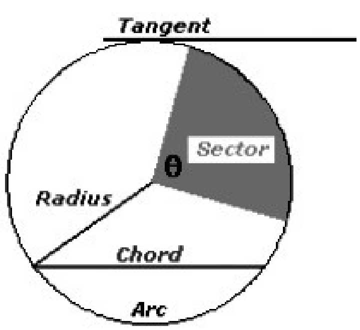
Regular Hexagon: A regular hexagon is a closed figure with six equal sides.



If we join each vertex to the centre of the hexagon, we get 6 equilateral triangles. Therefore, if the side of the hexagon is a, each equilateral triangle has a side a. Hence, area of the regular hexagon

$= 6\times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a2$

A circle is a set of all points in a plane that lie at a constant distance from a fixed point. The fixed point is called the center of the circle and the constant distance is known as the radius of the circle.



Arc: An arc is a curved line that is part of the circumference of a circle. A **minor arc** is an arc less than the semicircle and a **major arc** is an arc greater than the semicircle.

Chord: A chord is a line segment within a circle that touches 2 points on the circle.

Diameter: The longest distance from one end of a circle to the other is known as the diameter. It is equal to twice the radius.

Circumference: The perimeter of the circle is called the circumference. The value of the circumference = 2πr, where r is the radius of the circle.

Area of a circle: Area = π × (radius)2 = πr2.

Sector: A sector is like a slice of pie (a circular wedge).

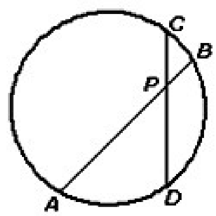
Area of Circle Sector: (with central angle θ) Area = $\frac{\theta}{360} \times \pi \times r^2$

Length of a Circular Arc: (with central angle θ) The length of the arc = $\frac{\theta}{360} \times 2\pi \times r$

Tangent of circle: A line perpendicular to the radius that touches ONLY one point on the circle

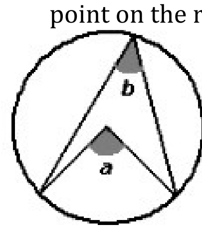
Equal chords are equidistant from the center.

Conversely, if two chords are equidistant from the center of a circle, they are equal. In the following figure, two chords of a circle, AB and CD, intersect at point P. Then, AP × PB = CP × PD.



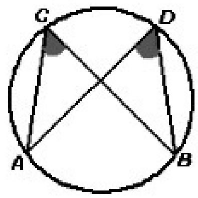
In a circle, equal chords subtend equal angles at the center.

The angle subtended by an arc of a circle at the center is double the angle subtended by it at any point on the remaining part of the circumference.



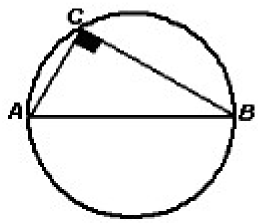
In the figure shown above, a = 2b.

Angles inscribed in the same arc are equal.



In the figure angle ACB = angle ADB.

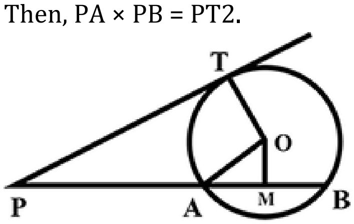
An angle inscribed in a semi-circle is a right angle.



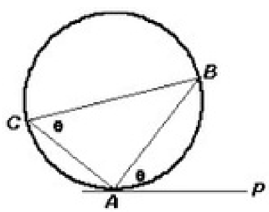
Let angle ACB be inscribed in the semi-circle ACB; that is, let AB be a diameter and let the vertex C lie on the circumference; then angle ACB is a right angle.

From an external point P, a secant P-A-B, intersecting the circle at A and B, and a tangent PT are drawn.

Then, PA × PB = PT2.

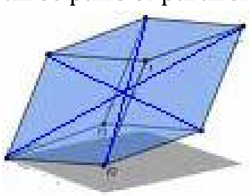


 The angle that a tangent to a circle makes with a chord drawn from the point of contact is equal to the angle subtended by that chord in the alternate segment of the circle.



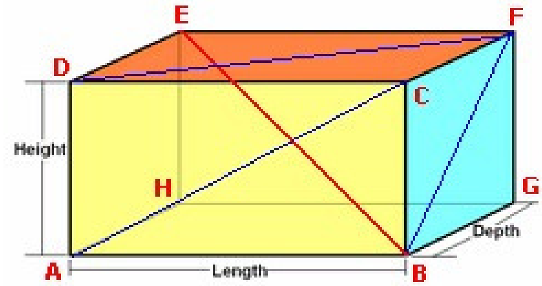
In the figure above, PA is the tangent at point A of the circle and AB is the chord at point A. Hence, angle BAP = angle ACB.

Parallelepiped: A parallelepiped is a solid bounded by three pairs of parallel plane faces.



- Each of the six faces of a parallelepiped is a parallelogram.
- Opposite faces are congruent.
- The four diagonals of a parallelepiped are concurrent and bisect one another.

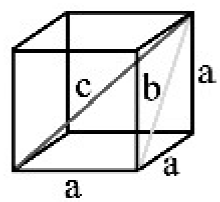
Cuboid: A parallelepiped whose faces are rectangular is called a cuboid. The three dimensions associated with a cuboid are its length, breadth and height (denoted as l, b and h here.)



- The length of the three pairs of face diagonals are = $\sqrt{b2+h2}$,$AC=\sqrt{l2+h2}$, and DF=$\sqrt{l2+b2}$.
- The length of the four equal body diagonals AF = $\sqrt{l2+b2+h2}$.
- The total surface area of the cuboid = 2(lb + bh + hl)
- Volume of a cuboid = lbh
- The radius of the sphere circumscribing the cuboid = $\frac{Diagonal}{2} = \frac{\sqrt{l2+b2\ h2}}{2}$
- Note that if the dimensions of the cuboid are not equal, there cannot be a sphere which can be inscribed in it, i.e. a sphere which touches all the faces from inside.

Euler's Formula: the number of faces (F), vertices (V), and edges (E) of a solid bound by plane faces are related by the formula F + V = E + 2 gives here 6 + 8 = 12 + 2.

Cube: A cube is a parallelepiped all of whose faces are squares.



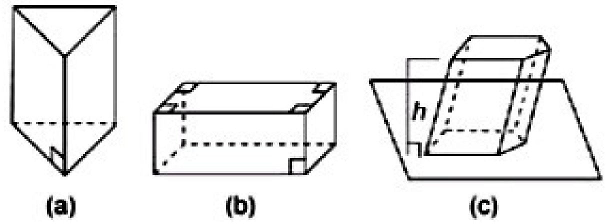
- Total surface area of the cube = 6a2
- Volume of the cube = a3
- Length of the face diagonal b = $\sqrt{2}a$
- Length of the body diagonal c = $\sqrt{3}a$
- Radius of the circumscribed sphere = $\frac{\sqrt{3}a}{2}$

Prism: A prism is a solid bounded by plane faces, of which two, called the ends, are congruent figures in parallel planes and the others, called side-faces are

Radius of the inscribed sphere = $\frac{a}{2}$

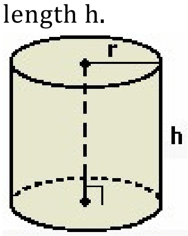
Radius of the sphere tangent to edges = $\frac{a}{\sqrt{2}}$

parallelograms. The ends of a prism may be triangles, quadrilaterals, or polygons of any number of sides.



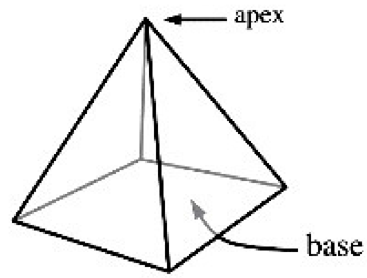
- The side- edges of every prism are all parallel and equal.
- A prism is said to be right, if the side-edges are perpendicular to the ends: In this case the side faces are rectangles. Cuboids and cubes are examples.
- Curved surface area of a right prism = perimeter of the base × height
- Total surface area of a right prism = perimeter of the base × height + 2 × area of the base
- Volume of a right prism = area of the base × height

aRight Circular Cylinder: A right circular cylinder is a right prism whose base is a circle. In the figure given below, the cylinder has a base of radius r and a height of length h.



- Curved surface area of the cylinder = 2πrh
- Total surface area of the cylinder = 2πrh + πr2
- Volume of the cylinder = πr2h

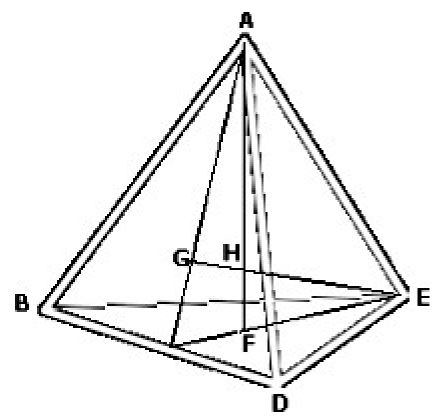
Pyramid: A pyramid is a solid bounded by plane faces, of which one, called the base, is any rectilinear figure, and the rest are triangles having a common vertex at some point not in the plane of the base.
The slant height of a pyramid is the height of its triangular faces. The height of a pyramid is the length of the perpendicular dropped from the vertex to the base.



In a pyramid with n sided regular polygon as its base,

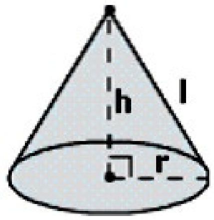
- Total number of vertices = n + 1
- Curved surface area of the pyramid = $\frac{Perimeter}{2} \times$ $slant\ height$
- Total surface area of the pyramid = $\frac{Perimeter}{2}$ × slant height + area of the base.
- Volume of the pyramid = $\frac{Base\ area}{3}$ ×height

Tetrahedron: A tetrahedron is a pyramid which has four congruent equilateral triangles as it four faces.
The figure below shows a tetrahedron with each face equal to an equilateral triangle of side a.



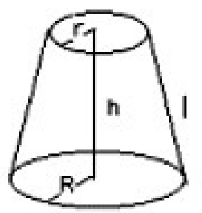
- Total number of vertices = 4 The four lines which join
- the vertices of a tetrahedron to the centroids of the opposite faces meet at a point which divides them in the ratio 3: 1. In the figure. AH: HF = 3: 1.
- Curved surface area of the tetrahedron = $\frac{3\sqrt{3}a2}{4}$
- Total surface area of the tetrahedron = $\sqrt{3}a2$.
- Height of the tetrahedron = $\frac{\sqrt{6}a}{3}$
- Volume of the tetrahedron = $\frac{\sqrt{2}a3}{12}$

Right Circular Cone: a right circular cone is a pyramid whose base is a circle. In the figure given below, the right circular cone has a base of radius r and a height of length h.



- Slant height $l = \sqrt{h^2+r^2}$
- Curved surface area of the cone = πrl
- Total surface area of the cone = $\pi rl + \pi r^2$
- Volume of the cone = $\frac{\pi r^2 h}{3}$

Frustum of a Cone: When a right circular cone is cut by a plane parallel to the base, the remaining portion is known as the frustum.



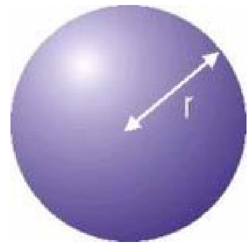
Slant height $l=\sqrt{h^2+(R-r)^2}$.

Curved surface area of the frustum = $\pi(r + R)l$

Total surface area = $\pi(r + R)l + \pi(r^2 + R^2)$

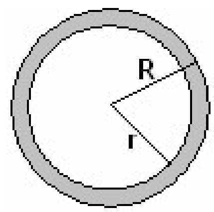
Volume of the frustum = $\frac{\pi h(r^2+R^2+R\,r)}{3}$

Sphere: A sphere is a set of all points in space which are at a fixed distance from a given point. The fixed point is called the centre of the sphere, and the fixed distance is the radius of the sphere.



- Surface area of a sphere = $4\pi r^2$
- Volume of a sphere = $\frac{4}{3}\pi r^3$

Spherical Shell: A hollow shell with inner and outer radii of r and R, respectively.



- Volume of the shell = $\frac{4}{3}\pi(R^3-r^3)$

PERMUTATIONS AND COMBINATIONS AND PROBABILITY

1. **PERMUTATIONS** If we are given n objects and we are supposed to arrange r (r ≤ n) objects out of these then the number of ways in which this can be done is written as nPr and is read as "the number of permutations of n objects taken r at a time".
   The number of permutations is given as:
   $^nP_r = \frac{n!}{(n-r)!}$
2. **PERMUTATIONS WITH REPETITIONS**
   If n objects are to be arranged among themselves and these contain p identical objects of one kind, q identical objects of another kind, r identical objects of still another kind and so on, the total number of ways in which they can be arranged is given by
   $\frac{n!}{p!q!r!..}$
3. **CIRCULAR ARRANGEMENTS**
   The number of ways in which n objects can be arranged in a circle = (n - 1)!

4. **DERANGEMENT**
   If the letters are to be put in the envelopes in such a way that none of the letters goes into the right envelope, the resulting kind of arrangement is called a **derangement.**
   Derangement is a permutation in which every element appears in the wrong position.
   The number of distinct derangements of n objects is
   $D_n = n!\sum_{i=0}^{n} \frac{(-1)^i}{i!}$
   $\therefore D_n = n!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots + \frac{(-1)^n}{n!}\right]$
5. **COMBINATIONS**
   If we are given n objects and we have to select r out of them (r ≤ n), then the number of ways in which this can be done is nCr and is read as "the number of combinations of n objects taken r at a time".
   The number of combinations is given as:
   $^nC_r = \frac{n!}{(n-r)!r!}$

6. **Some More Useful Rules Involving Combinations**

- The number of ways in which n identical things can be divided in r groups when each person may get any number of things, including zero = (n+r-1)C(r-1)
- The number of ways in which n identical things can be divided in r groups when each person may get at least one thing = (n-1)C(r-1)

7. **PROBABILITY**

Probability is the chance of occurrence of an event If S is the sample space of an unbiased experiment and if F is an event, then the probability that B takes place is

$$P(E) = \frac{n(E)}{n(S)}$$

8. **THE ADDITION THEOREM OF PROBABILITY** If A and B are events of a sample space S, then B) = n(A) + n(B) - n(A ∩ B) dividing the n(A ∪ by n(s), we have

$$\therefore P(A \cup B) = p(A) + p(B) - p(A \cap B)$$

9. **CONDITIONAL PROBABILITY**

Let the event that the first card drawn is a spade be called A, and let the event that the second card drawn is a spade be called B.

10. **REMEMBER:**

- If A and B are mutually exclusive events, P(A ∪ B) = P(A) + P(B)
- If A and B are exhaustive events, P(A) + P(B) = 1 and
- P(B) = 1 - P(A)

Odds in Favour $= \frac{Number\ of\ favourable\ cases}{Number\ of\ unfavourable\ cases}$

Odds Against $= \frac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}}$

If A and B are dependent events,

P(A ∩ B) = P(A).P(B/A), where P(A) ≠ 0

P(A ∩ B) = P(B).P(A/B), where P(B) ≠ 0

- If events A and B are independent, P(A ∩ B) = P(A) × P(B)